Models and numerical methods for geophysical flows

Application to sustainable energies.

J. Sainte-Marie
(**ANGE** - Numerical Analysis, Geophysics & Ecology)

also with

- N. Aissiouene, E. Audusse, M.-O. Bristeau, E. Godlewski,
- R. Hamouda, A. Mangeney, B. Perthame, N. Seguin

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Outline

3 aspects : modelling, analysis & simulation

- Incompressible Euler with free surface

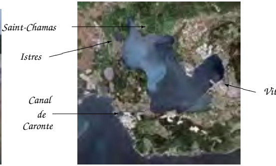
$$\nabla.\underline{\mathbf{u}} = 0$$
$$\frac{\partial \underline{\mathbf{u}}}{\partial t} + (\underline{\mathbf{u}}.\nabla)\underline{\mathbf{u}} + \nabla p = \mathbf{G}$$

  - represent various phenomena,
    more general/complex than Shallow Water equations
  - non smooth solutions i.e. with shocks
  - very difficult to analyse and simulate

- Gravity driven flows
  - fluids with complex rheology, debris flows, tsunami
  - couplings : erosion, hydrodynamics/biology, codes/models

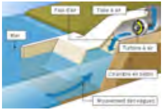
Gravity driven flows

- Hydraulic energy & dams
  - Malpasset 1959
  - ecology in reservoir



- Hazardous flows

- Marine energies
  - sea farm, buoys



- Fluid with complex rheology
  - avalanches, debris flow

Contents

Shallow water flows

- The cornerstone
- More sophisticated models needed

3d Navier-Stokes (Euler)

- Efficient/robust techniques exist
- Non-hydrostatic/dispersive models (very complex)

Renewable energies

- Marine energies
- Algae culture - hydrodynamics-biology coupling

Main ideas

- Not so hyperbolic systems
- Need for robust/efficient numerical schemes
- Towards optimization problems, data assimilation,. . .

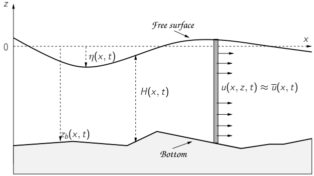
Shallow Water approximation of Navier-Stokes

$$(NS)\begin{cases} \operatorname{div}\underline{\mathbf{u}} = 0, \\ \dot{\underline{\mathbf{u}}} + (\underline{\mathbf{u}}.\nabla)\underline{\mathbf{u}} + \nabla p = \mathbf{G} + \operatorname{div}\underline{\underline{\Sigma}}, \end{cases}$$

- small parameter $\varepsilon = \frac{H_0}{L_0}$, Saint-Venant 1872,...

$$(SV)\begin{cases} \frac{\partial H}{\partial t} + \frac{\partial (H\bar{u})}{\partial x} = 0, \\ \frac{\partial (H\bar{u})}{\partial t} + \frac{\partial}{\partial x}\left(H\bar{u}^2 + \frac{g}{2}H^2\right) = -gH\frac{\partial z_b}{\partial x} \end{cases}$$

- Reduced complexity
- Many applications
- Hyperbolic CL

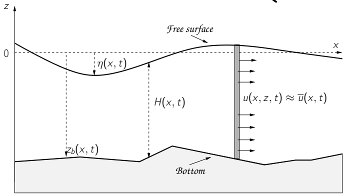


NOTATIONS: $\bar{u}(x,t) = \frac{1}{H}\int_{z_b}^{\eta} u(x,z,t)dz$

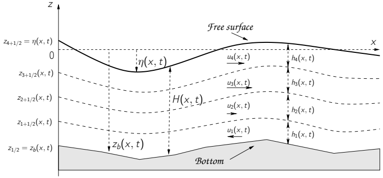
Some remaining difficulties arround the Shallow Water syst. but scientific challenges and promising applications concern more complex models.

Incompressible hydrostatic Euler (NS) system

$$\left\{\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0\\ &\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uw}{\partial z} + \frac{\partial p}{\partial x} = 0\\ &\frac{\partial p}{\partial z} = -g \end{aligned}\right.$$



$\Rightarrow$



- Beyond the Saint-Venant system
- Many contributions (Audusse, Bouchut, LeVeque, Pares, ...)
- Isopycnal models / non-miscible fluids / sigma-transform

Key idea

Saint-Venant
$u(x,z,t) \approx \bar{u}(x,t)$

$\Rightarrow$

"Multilayer" Saint-Venant
$u(x,z,t) \approx \sum_{\alpha=1}^{N} \mathbf{1}_{z\in L_\alpha(x,t)} u_\alpha(x,t)$

Vertical discretization (M$_2$AN, 2011)

The hydrostatic Euler system

$\rho = \mathbb{1}_{z_b \le z \le \eta}$

$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho w}{\partial z} = 0 \\ \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u w}{\partial z} + \rho \frac{\partial p}{\partial x} = 0 \\ \frac{\partial p}{\partial z} = -\rho g \end{array} \right.$$

Weak form ($\mathbb{P}_0^t$) of the Euler system

with $\mathbb{P}_0^t = \left\{ \mathbb{1}_{z \in L_\alpha(x,t)}(z),\ 1 \le \alpha \le N \right\}$ $\langle f \rangle_\alpha = \int_{z_b}^{\eta} f \, \mathbb{1}_{z \in L_\alpha(x,t)}(z)\, dz$

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} \langle \rho \rangle_\alpha + \frac{\partial}{\partial x} \langle \rho u \rangle_\alpha = G_{\alpha+1/2} - G_{\alpha-1/2} \\ \frac{\partial}{\partial t} \langle \rho u \rangle_\alpha + \frac{\partial}{\partial x} \left(\langle \rho u^2 \rangle_\alpha + \frac{g}{2} \langle \rho (z - z_b) \rangle_\alpha \right) \\ \qquad = u_{\alpha+1/2} G_{\alpha+1/2} - u_{\alpha-1/2} G_{\alpha-1/2} \end{array} \right.$$

Energy equation $\frac{\partial}{\partial t} \langle \rho E \rangle_\alpha + \frac{\partial}{\partial x} \langle \rho u (E + p) \rangle_\alpha = E_{\alpha+1/2} - E_{\alpha-1/2}$

Closure relations needed

Closure relations & Galerkin approx.

Similar to moment closure [Levermore]

Energy balance

$$\frac{\partial}{\partial t}\langle \rho E\rangle_\alpha + \frac{\partial}{\partial x}\langle \rho u(E+p)\rangle_\alpha = E_{\alpha+1/2} - E_{\alpha-1/2}$$

with $E = \frac{u^2}{2} + gz$

- deviation w.r.t. depth-averages

$$u = \frac{\langle \rho u\rangle_\alpha}{\langle \rho\rangle_\alpha} + u'_\alpha,$$

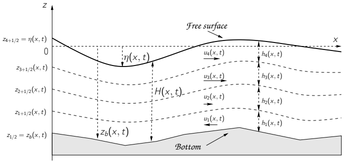
Minimization problem

$$\langle \rho E\left(z; \frac{\langle \rho u\rangle_\alpha}{\langle \rho\rangle_\alpha}, \frac{\langle \rho w\rangle_\alpha}{\langle \rho\rangle_\alpha}\right)\rangle_\alpha = \min_{u',w'} \langle \{\rho E(z; u, w)\}\rangle_\alpha$$

Leading to

$$\langle \rho u^2\rangle_\alpha = \frac{\langle \rho u\rangle_\alpha^2}{\langle \rho\rangle_\alpha}$$

The obtained model (M$_2$AN 2010, JCP 2011)



- Galerkin approx. of the Euler system over $\mathbb{P}_0^t = \left\{\mathbb{1}_{z\in L_\alpha(x,t)}(z),\ 1 \le \alpha \le N\right\}$ with minimal energy

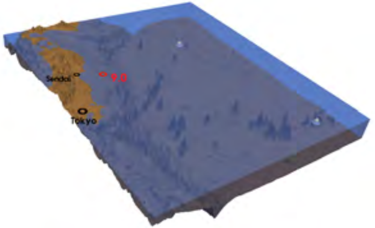
$$\left\{\begin{array}{l}\frac{\partial H}{\partial t} + \sum_{\alpha=1}^{N} \frac{\partial}{\partial x}(h_\alpha u_\alpha) = 0\\ \frac{\partial (h_\alpha u_\alpha)}{\partial t} + \frac{\partial}{\partial x}\left(h_\alpha u_\alpha^2 + \frac{g}{2} h_\alpha H\right) = u_{\alpha+1/2} G_{\alpha+1/2} - u_{\alpha-1/2} G_{\alpha-1/2}\\ \frac{\partial E_\alpha}{\partial t} + \frac{\partial}{\partial x}\left(u_\alpha \left(E_\alpha + \frac{g}{2} h_\alpha H\right)\right) = E_{\alpha+1/2} - E_{\alpha-1/2}\end{array}\right.$$

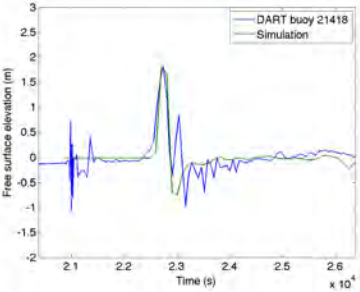
- Only one "global" continuity equation, $H = \sum h_\alpha$
- Exchange terms $G_{\alpha+1/2}$, $F_{\alpha+1/2} = u_{\alpha+1/2} G_{\alpha+1/2} + P_{\alpha+1/2}$,

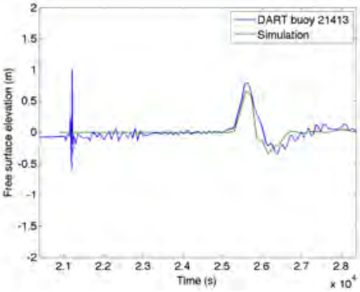
$$E_{\alpha+1/2} = \frac{u_{\alpha+1/2}^2}{2} G_{\alpha+1/2} + u_{\alpha+1/2} P_{\alpha+1/2}$$

- If $G_{\alpha+1/2} \equiv 0$ non-miscible fluids, N cont. equations

Typical applications (I)







Interest of simulations (mediteranean sea)

Typical applications (II)

- Hydrodynamics-biology coupling in a raceway

- Variable density flows

$$\begin{cases} \dot{\rho} + \mathrm{div}\,(\rho\underline{\mathbf{u}}) = 0, \\ \dot{\overline{\rho\underline{\mathbf{u}}}} + (\underline{\mathbf{u}}.\nabla)(\rho\underline{\mathbf{u}}) + \nabla p = \rho\mathbf{G}, \\ \dot{\overline{\rho T}} + \mathrm{div}\,(\rho T\underline{\mathbf{u}}) = \mu_T \Delta T, \end{cases}$$

$$\text{with } \rho = \rho(T,S) \qquad (= \rho(T,S,H))$$

Towards non-hydrostatic models

$$\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial w^2}{\partial z} + \frac{\partial p}{\partial z} = -g \quad \Rightarrow \quad \frac{\partial p}{\partial z} = -g$$

- No more compressible fluid mechanics
- Illustrations
  - (3 models)
  - (Hydro vs. non-hydro)
  - (Flow over a bump)
  - (Wave over a beach)

- An open problem - Many difficulties
  - several models/results (Peregrine, Bona, Saut, Lannes, Gavrilyuk, JSM,. . .)
  - only hyperbolic features, instabilities
  - often costly/unstable algorithms

The proposed model (DCDS 2014)

- Saint-Venant + non-hydrostatic terms

$$\left\{\begin{array}{l}\frac{\partial H}{\partial t}+\frac{\partial(H\bar{u})}{\partial x}=0\\ \frac{\partial(H\bar{u})}{\partial t}+\frac{\partial}{\partial x}\left(H\bar{u}^2+\frac{g}{2}H^2+H\overline{p}_{nh}\right)=-(gH+2\overline{p}_{nh})\frac{\partial z_b}{\partial x}\\ \frac{\partial(H\bar{w})}{\partial t}+\frac{\partial(H\bar{w}\bar{u})}{\partial x}=2\overline{p}_{nh}\\ \bar{w}=-\frac{H}{2}\frac{\partial\bar{u}}{\partial x}+\frac{\partial z_b}{\partial x}\bar{u}\end{array}\right.$$

  - rigourous derivation, energy balance
  - kinetic interpretation $M=\frac{H}{c}\chi_1\left(\frac{\xi-\overline{u}}{c}\right)\delta\left(\gamma-\overline{w}\right)$
  - a family of analytical solutions

- Energy equality (smooth solutions)

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x}\left(\overline{u}\big(E+\frac{g}{2}H^2+\frac{H}{2}\overline{p}_{nh}\big)\right)=0,$$

with $E=\frac{H(\overline{u}^2+\overline{w}^2)}{2}+\frac{gH(\eta+z_b)}{2}$

Summary of the 1^{st} part

- 3d Navier-Stokes (Euler) equations with free surface
  - non hydrostatic flows remains difficult to treat/discretize
  - approximated models exist for hydrostatic flows

☐ Good approximations or not ?

☐ Numerical analysis and discretization

Kinetic description

- A fantastic tool for
  - mathematical analysis
  - physical understanding
  - numerical analysis and schemes
- Basis : adopt a microscopic description (Boltzmann)

$$(Cont.\ model) \quad \frac{\partial X}{\partial t} + \frac{\partial F(X)}{\partial x} = 0 \quad \Leftrightarrow \quad \frac{\partial M}{\partial t} + \xi \frac{\partial M}{\partial x} = Q(x, t, \xi)$$

  - $M(x, t, \xi)$ particle density, $Q(x, t, \xi)$ collision term ($= 0$ a.e.)
  - $\int_{\mathbb{R}} \xi^p M \, d\xi$ gives the macroscopic variables
  - linear transport equation
- Only kinetic representations and not kinetic formulations

Properties of the Saint-Venant system

$$(SV)\begin{cases} \frac{\partial H}{\partial t} + \frac{\partial (H\bar{u})}{\partial x} = 0, \\ \frac{\partial (H\bar{u})}{\partial t} + \frac{\partial}{\partial x}\left(H\bar{u}^2 + \frac{g}{2}H^2\right) = -gH\frac{\partial z_b}{\partial x} \end{cases}$$

- An hyperbolic system with source term
- Domain invariant $H \geq 0$
- It admits a convex entropy (the energy)

$$\frac{\partial \overline{E}}{\partial t} + \frac{\partial}{\partial x}\overline{u}\left(\overline{E} + \frac{g}{2}H^2\right) \leq 0$$

- Kinetic interpretation

$$(SV) \quad \Leftrightarrow \quad \frac{\partial M}{\partial t} + \xi\frac{\partial M}{\partial x} - g\frac{\partial z_b}{\partial x}\frac{\partial M}{\partial \xi} = Q(x,t,\xi)$$

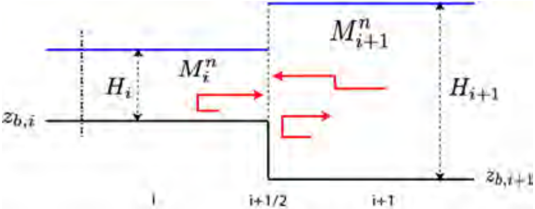
  - Gibbs equilibrium $M(x,t,\xi) = \frac{H}{c}\chi\left(\frac{\xi - \bar{u}}{c}\right)$
  - Macroscopic variables $(H, H\bar{u}, \bar{E}) = \int_{\mathbb{R}}(1,\xi,\xi^2/2)M\, d\xi$

Numerical methods

$$\frac{\partial M}{\partial t} + \xi \frac{\partial M}{\partial x} - g \frac{\partial z_b}{\partial x} \frac{\partial M}{\partial \xi} = Q(x,t,\xi)$$

$$\Downarrow$$

$$M_{i+}^{n+1-} = M_i^n - 2\frac{\Delta t^n}{\Delta x_i} \left(\xi M_{i+1/2}^n - \xi M_i^n + (\xi - u_i^n)(M_{i+1/2-}^n - M_i^n) \right)$$



$\Rightarrow$ positivity, well-balancing, consistency, discrete entropy

Theorem 1 The hydrostatic reconstruction is entropy-satisfying

Theorem 2 Convergence of the scheme (with F. bouchut)

The hydrostatic Euler (NS) system

$$\left\{\begin{array}{l}\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0\\ \frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uw}{\partial z}+\frac{\partial p}{\partial x}=0\\ \qquad\frac{\partial p}{\partial z}=-g\\ +\text{ BC (free surface)}\end{array}\right.$$

Approximated by

$$\left\{\begin{array}{l}\frac{\partial H}{\partial t}+\sum_{\alpha=1}^{N}\frac{\partial}{\partial x}(h_\alpha u_\alpha)=0\\ \frac{\partial(h_\alpha u_\alpha)}{\partial t}+\frac{\partial}{\partial x}\left(h_\alpha u_\alpha^2+\frac{g}{2}h_\alpha H\right)=u_{\alpha+1/2}G_{\alpha+1/2}-u_{\alpha-1/2}G_{\alpha-1/2}\\ \frac{\partial E_\alpha}{\partial t}+\frac{\partial}{\partial x}\left(u_\alpha\left(E_\alpha+\frac{g}{2}h_\alpha H\right)\right)=E_{\alpha+1/2}-E_{\alpha-1/2}\end{array}\right.$$

Two questions

- How to discretize it ?
- Is it a good approximation of the Euler (NS) system ?

Properties of the model

- **Hyperbolicity ?**

$$M(X)\dot{X} + F(X)\frac{\partial X}{\partial x} = 0$$

  - "often" hyperbolic, hyperbolic for $N = 2$
  - for $N > 2$, "arrow matrices" and interlacing of eigenvalues

$$\frac{1}{N}\sum_{1}^{N} u_i^2 \leq gH \qquad (\textit{generalized Froude number})$$

  - large family of entropies

- When $N \to \infty$?
- Proposition *The functions $(h_\alpha, u_\alpha, E_\alpha)(t, x)$ are strong solutions of MSV iff $\{M_j(x, t, \xi)\}_{j=1}^{N}$ is solution of*

$$\frac{\partial M_\alpha}{\partial t} + \xi\frac{\partial M_\alpha}{\partial x} - N_{\alpha+1/2} + N_{\alpha-1/2} = Q_\alpha(x, t, \xi)$$

- robust & efficient numerical scheme

Properties of the scheme

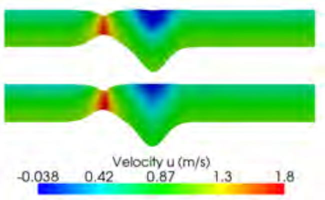
- Consistency
- Positivity
- Well-balancing
- Semi (fully) discrete entropy inequality
  - valid for non smooth solutions

Analytical validation (CMS 2012)

- Analytical solutions to the Euler system
  - 2d and 3d solutions, for any bottom topography $z_b(x,y)$
  - with entropic shocks
  - not necessarily free surface flows
  - In 2d (continuous solutions), u and H characterized by

$$u = \alpha\beta\frac{\cos\beta(z-z_b)}{\sin\beta H}, \qquad \left(g(H+z_b) + \frac{\alpha^2\beta^2}{2\sin(\beta H)^2}\right)_x = 0$$

- Recovered by the 2d and 3d codes (also without free surface)
  - analytic
  - simulated



- Fist and second order convergence (space and time)

Summary of the 2^{nd} part

- 3d Navier-Stokes (Euler) equations with free surface
  - non hydrostatic flows remains difficult to treat/discretize
  - approximated models exist for hydrostatic flows

- Rather good approximations ?
  - analytical validations
  - experimental validations

- Numerical analysis and discretization
  - "simple" schemes
  - stable/robust schemes

□ What to do with such models ?
  - Control / Data assimilation
  - Marine energies

Control/DA : interest of the kinetic description

Macroscopic level | **Kinetic level**

$$\frac{\partial X^{obs}}{\partial t} + \nabla . F(X^{obs}) = 0 \quad \Leftrightarrow \quad \begin{cases} \frac{\partial M^{obs}}{\partial t} + \xi \frac{\partial M^{obs}}{\partial x} = Q \\ X^{obs} = \int_{\mathbb{R}} (1, \xi, \xi^2/2) M^{obs} \, d\xi \end{cases}$$

$$\Downarrow \qquad \Downarrow$$

$$\begin{cases} \frac{\partial X}{\partial t} + \nabla . F(X) = \lambda(X^{obs} - X) \\ X(0) = \bar{X}_0 + \delta_0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} \frac{\partial f}{\partial t} + \xi \frac{\partial f}{\partial \underline{x}} = \lambda(M^{obs} - f) \\ f(0) = \bar{M}_0 + \gamma_0 \end{cases}$$

$$X \overset{?}{\to} X^{obs} \qquad f \overset{?}{\to} M^{obs}$$

- very difficult
- L_1 contraction
- geometry

- $f = \int_0^t M^{obs}\big(a, x - \xi(t-a), \xi\big) e^{-\lambda(t-a)} da$
- BGK type model
- partial observations

Control/DA : two examples

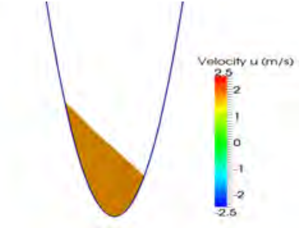
Main result

Run:WaveDAT

- proof of the convergence

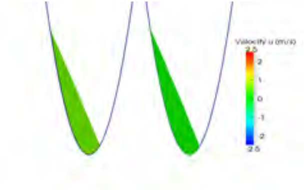
Thacker analytical solution

- Stable numerical scheme required
- Drying/flooding phenomena
- Space & time scheme



Data assimilation

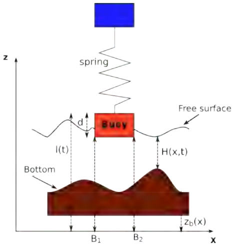
- Measurements of the water depth
- 200 obs. vs. 200 000 time steps
- 20% of the domain

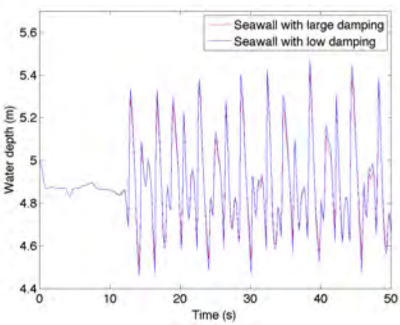


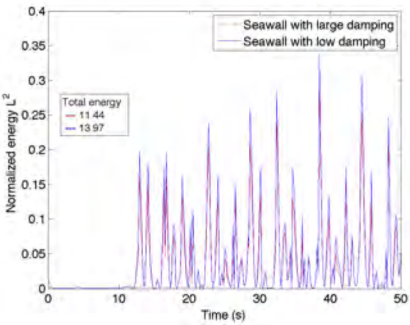
Capture wave energy

- Collaboration with Openocean
- An experimental device
  (Pecem harbor - Ceara - Brazil)

Obtained results

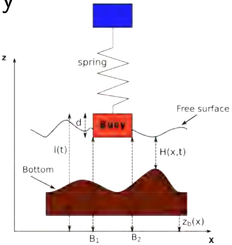






Towards optimization

- A buoy / a topography



- A simplified/realistic model for wave propagation (Run:3_models)

$$\frac{\partial^2 \eta}{\partial t^2} - \nabla.\left(g H_0 \nabla \eta\right) = 0$$

- A cost function

$$\text{argmin }_{z_b} J(\eta, z_b) \quad \text{with} \quad J(\eta, z_b) = \int_{t_0}^{T} \left(\left.\frac{\partial \eta}{\partial t}\right|_{x=x_0} \right)^2 dt$$

Microalgae and biofuel production

Description : algae pool driven into motion by a paddle wheel

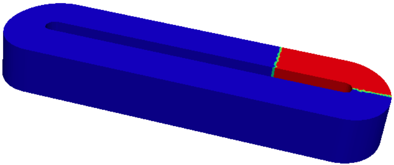
Companies : EADS, Microphyt, Naskéo, Ondalys, Roquette, Sofiprotéol, Soliance,. . .

Goal : optimize the biomass production by playing on the nutrients supply, water depth, agitation,. . .

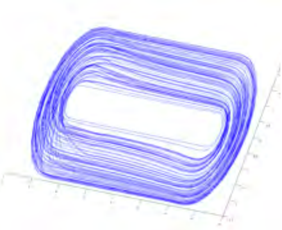
Production : lipids (methanisation), CH_4, biofuel

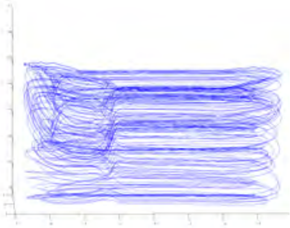
An optimization problem coupling hydrodynamics and biology

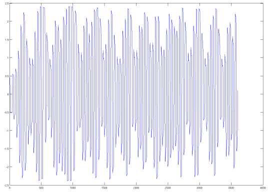
Illustration



3d motion (anim)







Fluid mechanics + biology

- Navier-Stokes with free surface

$$\left\{\begin{array}{l}\operatorname{div}\underline{\mathbf{u}} = 0,\\ \dot{\underline{\mathbf{u}}} + (\underline{\mathbf{u}}.\nabla)\underline{\mathbf{u}} + \nabla p = \mathbf{G} + \operatorname{div}\underline{\underline{\Sigma}},\end{array}\right.$$

- Droop model (Droop 1983)

$$\left\{\begin{array}{l}\frac{dC_1}{dt} = \mu(\frac{C_2}{C_1}, I)C_1 - RC_1\\ \frac{dC_2}{dt} = -\lambda(C_3, \frac{C_2}{C_1})C_1\\ \frac{dC_3}{dt} = \lambda(C_3, \frac{C_2}{C_1})C_1 - RC_2\end{array}\right.$$

with C_1 : phytoplanktonic carbon, C_2 residual nitrates and C_3 phytoplanktonic nitrogen, I light, R death rate

- Advection, reaction and diffusion PDE's

$$\frac{\partial X}{\partial t} + \nabla.(\underline{\mathbf{u}}X) = F(X) + \nu\Delta X$$

with $X = (C_1, C_2, C_3)^T$

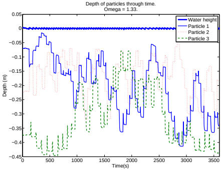
An optimization problem

$$\max P_{algae} \quad vs. \quad \min E_{wheel}$$

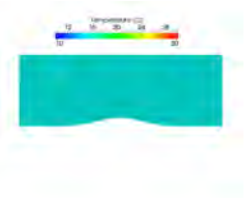
Playing on

- Water level, nutrients
- Geometry, algae species
- Lagrangian trajectories

$$\frac{dM}{dt} = \underline{u}(M(t))$$



- Solar driven flows



Conclusion

- Towards Euler (Navier-Stokes) system
  - OK for hydrostatic flows
  - non-hydrostatic models. . . a lot of works

- Analysis results
  - very difficult
  - on going works (with the help of B. Perthame)

- Need for robust/efficient numerical schemes
  - 2d, 3d
  - high order schemes, compromise between stability/accuracy

- Towards control, optimization, FSI
  - difficult but very interesting applications

Taiwan wine

Le vin taiwanais Putao médaillé d'or en France

Publié le 13 mars 2014 par megwang

Chen Chienhao, Muscat doré et Taichung! Voici une combinaison à la taiwanaise réussie qui a permis au pays de décrocher pour la toute première fois une médaille d'or dans le cadre des Vinalies Internationales. Fondées en 1993 par l'Union des œnologues de France, les Vinalies internationales, agréées par l'OIV (Organisation internationale de la vigne et du vin), accueillent une centaine de juges du monde de l'œnologie. Cette rencontre annuelle est organisée conformément au règlement des concours internationaux définis par l'OIV. Cette année, 141 juges de différentes nationalités ont sélectionné les meilleurs vins de sept catégories : vin rouge sec, vin rosé sec, vin blanc sec, vin liquoreux, liqueur, vin effervescent et eau-de-vie.

(http://rtifrench.files.wordpress.com/2014/03/photo-1.jpg)
Vin Putao, médaille d'or de Vinalies Internationales 2014

Dans le cadre de cette compétition, le muscat doré du Domaine Shu Sheng du millésime 2007 vient de remporter le médaille d'or des Vinalies internationales de la catégorie vin liquoreux. Ce vin produit officiellement pour la première fois en 2009 dans le centre de Taiwan a été développé par Chen Chienhao, à la fois, œnologue, producteur de vin et également sommelier formé en France. Compte tenu du climat chaud et humide de Taiwan, il s'est inspiré de la méthode de production du vin portugais Madère en installant une serre à énergie solaire dans le but de contrôler la température durant la maturation du vin. Pour la production de ce vin au nom de Putao (l'homonyme de « raisin » en Chinois), Chen Chienhao a séléctionné le Moscato Oro, un cépage bien sucré que les Taiwanais ont introduit dès l'époque japonaise au début du XXème siècle. En 2010, deux cents bouteilles de Putao ont été produites et il a été immédiatement sélectionné par le restaurant français Pasadena dans le sud de Taiwan pour accompagner le dessert "Croquant de pamplemousse" cuit et cru du restaurant parisien Pavillon Ledoyen.

(http://rtifrench.files.wordpress.com/2014/03/photo-2.jpg)
Chen Chienhao, l'homme qui souhaite initier l'histoire du vin à Taiwan avec son Putao

Aujourd'hui, le vin Putao est produit entre 600 et 700 bouteilles par an. Cependant, il faut attendre au minimum 2017 pour avoir la chance de le déguster puisque la production des deux prochaines années est déjà toute réservée sur le carnet de commandes du viticulteur!